

## Exercise 60

The *biomass*  $B(t)$  of a fish population is the total mass of the members of the population at time  $t$ . It is the product of the number of individuals  $N(t)$  in the population and the average mass  $M(t)$  of a fish at time  $t$ . In the case of guppies, breeding occurs continually. Suppose that at time  $t = 4$  weeks the population is 820 guppies and is growing at a rate of 50 guppies per week, while the average mass is 1.2 g and is increasing at a rate of 0.14 g/week. At what rate is the biomass increasing when  $t = 4$ ?

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### Solution

If the population at  $t$  weeks is  $N(t)$ , then

$$N(4) = 820$$

and

$$N'(4) = 50.$$

If the average mass at  $t$  weeks is  $M(t)$ , then

$$M(4) = 1.2$$

and

$$M'(4) = 0.14.$$

The biomass is the product of the population and the average mass.

$$B(t) = N(t)M(t)$$

The rate at which the biomass increases with respect to time is  $B'(t)$ . Use the product rule to evaluate the derivative.

$$B'(t) = \frac{d}{dt}[N(t)M(t)] = N'(t)M(t) + N(t)M'(t)$$

At  $t = 4$  weeks, it is

$$B'(4) = N'(4)M(4) + N(4)M'(4) = (50)(1.2) + (820)(0.14) = 174.8 \frac{\text{g}}{\text{week}}.$$